TECHNICAL REPORT ARLCB-TR-83016

EVALUATION OF SUPERCONDUCTING AUGMENTATION ON A RAIL GUN SYSTEM

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JUNE 1983

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARLCB-TR-83016	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) EVALUATION OF SUPERCONDUCTING AUGMENTATION ON A RAIL GUN SYSTEM		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) C. G. Homan and W. Scholz		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Armament Research & Development Command Benet Weapons Laboratory, DRDAR-LCB-TL Watervliet, NY 12189		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMCMS No. 6111.02.H600.011 Pron No. 1A325B541A1A
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Armament Research & Development Command Large Caliber Weapon Systems Laboratory Dover, NJ 07801		12. REPORT DATE June 1983
		13. NUMBER OF PAGES 24
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Presented at 4th International Pulsed Power Conference,Albuquerque,NM. June 1983.Published in proceedings of the Second Symposium on Electromagnetic Launch Technology

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Rail Gun
Normally Conducting Augmentation
Superconducting Augmentation
System Comparison
Launch Efficiency
Constant Rail Current
Variable Rail Current

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The simple rail gun and rail guns with normally conducting and superconducting augmentation are discussed using an energy approach. Ideal launch efficiencies neglecting Joule losses and assuming constant rail current during the launch are shown to be 50 percent for normally conducting systems, and up to 100 percent for systems with superconducting augmentation. Energy requirements of an actual system are compared with expected values for a system with superconducting augmentation. The situation of variable rail currents has also been discussed.

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ACKNOWLEDGEMENTS

The authors would like to thank T. Gora (DRDAR-LCA) for his careful reading of the manuscript.

INTRODUCTION

This report will compare the effects of augmentation coils on a simple rail gun system using both superconducting and normal conductors for the augmentation coil(s).

The use of superconducting augmentation introduces two additional factors into the energy management of a rail gun system. These factors contribute to the time management of maintaining the augmentation field in the system and result from the physical facts that, unlike magnets wound from normal metals, superconducting coils experience no Joule losses and have the property of magnetic flux exclusion. Both factors will have effects on the energy management in a superconducting rail gun system resulting in significant savings in energy lost compared to the operation of a completely normal conducting system.

This evaluation of augmentation will be made for ideal systems and an actual system using the design parameters proposed for an electromagnetic air defense gun.[1]

THE AUGMENTED RAIL GUN

Figure 1 shows schematically the augmented rail gun configuration. The box labeled energy source contains all the components of the system required to provide properly pulsed energy (current) to the slider rails creating the magnetic field which transmits the mechanical energy to the projectile via

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[1] McNab, I. R. and Deis, D. W., "Study of an Electromagnetic Gun System For Air Defense," Westinghouse Research Center, November 1981. Final report prepared for the Air Force Armament Laboratory, Air Force Report No. AFATL-TR-81-99.

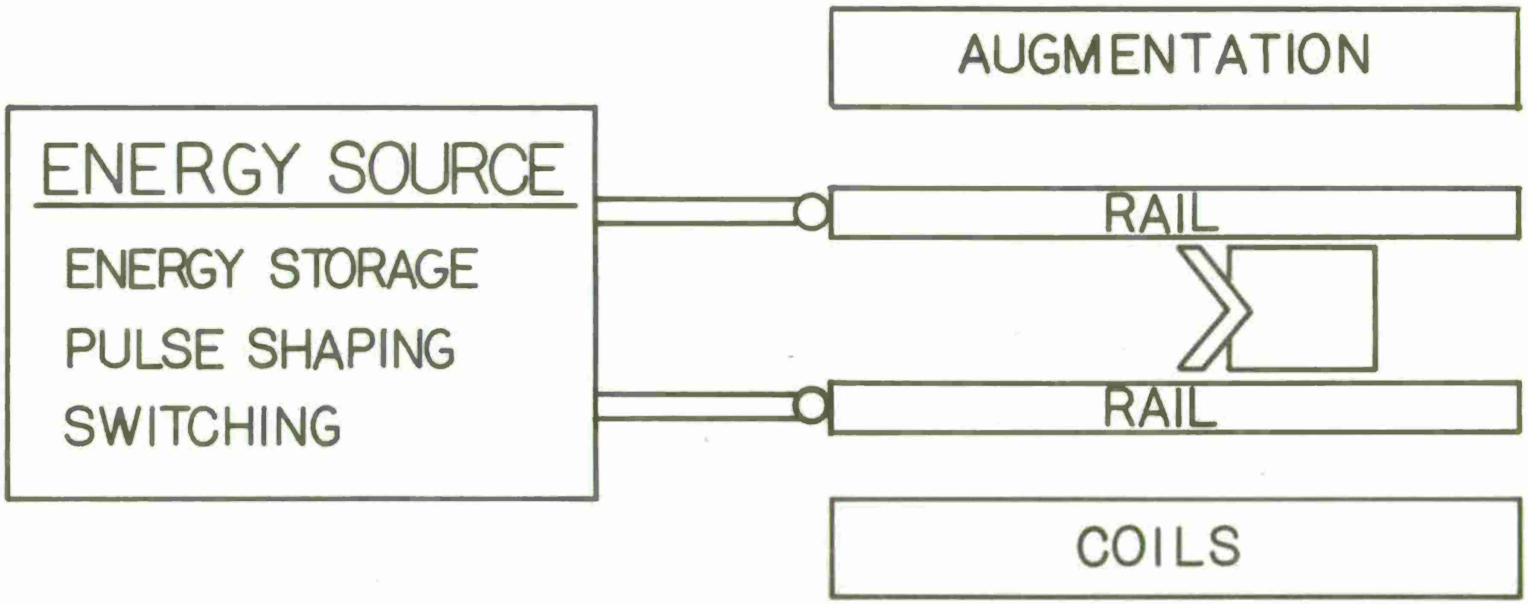

Figure 1. The Augmented Rail Gun Configuration.

Lorentz forces. The expression for the Lorentz force in a simple rail gun without augmentation is[2]

$$F_{EM} = \frac{1}{2} L' I^2 \tag{1}$$

where L' is the inductance per unit length of the rails and I is the current in the rails.

Augmentation provides an additional field which enhances the energy transfer to a projectile. Since the augmentation field exists both in front of as well as in back of the projectile, it is twice as effective as the rail field itself.[3]

ENERGY ANALYSIS OF IDEAL RAIL GUN CONFIGURATIONS

In this analysis we will assume that the geometries of the configurations are fixed, that is, suitable support of the rails and augmentation coil(s) is provided in the design to prevent mechanical energy conversion into deformation energy in both systems.

The Joule heating losses will not be analyzed specifically since the design of the rails and the normally conducting augmentation coil(s) strongly affect this loss. In a later section, we will point out some fundamental savings in Joule losses which can be achieved using superconducting augmentation coils, but in this section we will make the assumption that Joule losses can be neglected.

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[2]McNab, I. R. and Deis, D. W., "Study of an Electromagnetic Gun System For Air Defense," Westinghouse Research Center, November 1981. Final report prepared for the Air Force Armament Laboratory, Air Force Report No. AFATL-TR-81-99, p. 7.

[3]Kolm, H. H., "Electromagnetic Guns," JANNAF Propulsion Mtg., Vol. 1, 1979.

Simple Rail Gun

In this case the energy source delivers a current I to the rails. Assuming a constant current I for the launch, the energy storage in the magnetic field W_m is

$$W_m = \int dW_m = \frac{1}{2} I^2 \int_0^L dL = \frac{1}{2} I^2 L \qquad (2)$$

where L is the self inductance of the rail circuit at the end of the launch, i.e., when the projectile leaves the rails. The assumption of constant current during launch simplifies the arguments to be made here. An analysis of the system including the driving coil inductance of the source for variable currents will be made in the Appendix following the procedures described in Reference 4. This analysis shows that the constant current assumption does not significantly affect the conclusions reached in this analysis.

The differential magnetic flux generated in this system $d\phi = I\, dL$ can now be used to calculate the work done by the energy source, W_s, in providing the magnetic field energy and the mechanical work on the projectile. That is

$$W_s = \int dW_s = I \int (d\phi/dt)dt = I \int_0^L I\, dL = I^2 L \qquad (3)$$

The mechanical work W_M provided the projectile (neglecting frictional, etc. losses) is

$$W_M = W_s - W_m = \frac{1}{2} I^2 L \qquad (4)$$

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[4]McNab, I. R. and Deis, D. W., "Study of an Electromagnetic Gun System For Air Defense," Westinghouse Research Center, November 1981. Final report prepared for the Air Force Armament Laboratory, Air Force Report No. AFATL-TR-81-99, Appendix A, p. 137.

It is important to note that the magnetic energy W_m stored in the rail field at the end of the launch is lost by dissipation in the muzzle resistor.

Rail Gun With Normal Conducting Augmentation Coils

In this case, the energy source must provide a constant current in both the rails I and a constant current in the augmentation coils I_A during the launch.

The change in energy stored in the magnetic fields (neglecting the magnetic energy stored in the self inductance field of the augmentation coils, which remains constant throughout the process and does not contribute to the launch energies in this case) is

$$W_m = \int dW_m = \frac{1}{2} I^2 \int_0^L dL + II_A \int_0^M dM = \frac{1}{2} I^2 L + II_A M \tag{5}$$

where L and M are, respectively, the self and the mutual inductance of the circuit at the end of the launch, i.e., where $L = \ell L'$ and $M = \ell M'$, ℓ is the length of the rails, and L' and M' are the self and mutual inductance per unit length.

Again the energy sources (rail and augmentation circuit) must provide both the magnetic field energy and the mechanical energy to the projectile. Since the differential magnetic flux through the rail circuit and the augmentation coils in this case is

$$d\phi = I\,dL + I\,dM + I_A\,dM \tag{6}$$

the work performed by the energy sources can be easily shown to be

$$W_s = I^2 L + 2II_A M \tag{7}$$

The mechanical work W_M is as before,

$$W_M = \frac{1}{2} I^2 L + II_A M \tag{8}$$

As in the case of the simple rail gun, the mechanical work equals the energy stored in the magnetic field. The latter is again dissipated by Joule heating after the projectile leaves the rails and is therefore lost. The advantage of normal conducting augmentation coils appears to be that a greater energy can be imparted to the projectile at the same current output from the rail energy source at the expense of an increased total power consumption. Of course, Joule heating losses will be greater in the system since magnetic fields cannot be maintained without losses in the normal conductors used for augmentation.

Rail Gun With Superconducting Augmentation Coils

Since a superconducting coil will retain its stored energy without loss, the self inductive magnetic energy may be extracted from the energy source before a projectile is launched. As we will show subsequently, this stored energy remains in the coil after the projectile is launched, except for small losses due to fluxon motion, as long as the coil is maintained in the superconducting state.

Initially, therefore, a stored energy in the self inductance field of the superconducting coil exists equal to $(1/2)I_{so}{}^2L_s$, where I_{so} is the superconducting current and L_s is the self inductance of the superconducting coil.

As the projectile is launched by a constant current through the rails, I, the change in magnetic field energy is

$$dW_m = \frac{1}{2} I^2 \, dL + II_s \, dM + IM \, dI_s + L_s I_s \, dI_s \tag{9}$$

where we note the current in the superconducting coil I_s varies as a function of I, L_s, and M, with $M = M'x$ where x is the displacement along the rail.

To evaluate this variation in supercurrent, we invoke the unique superconducting property of zero resistance. That is, using Faraday's law, there can be no induced emf in a superconducting coil.

$$\oint \underline{E} \cdot d\underline{l} = - d\phi /dt = 0 \qquad \text{(superconducting coil)} \tag{10}$$

The flux threading the superconducting coil is constant and in differential form is

$$d\phi = L_s \, dI_s + I \, dM = 0 \tag{11}$$

assuming a constant current from the energy source during launch and constant self inductance L_s of the superconducting coil.

Substituting Eq. (11) in Eq. (9) and integrating, we obtain for the energy of the magnetic field W_m (omitting the constant of integration $(1/2)L_sI_{so}{}^2$ here and in the work required by the energy source since it is recovered).

$$W_m = \frac{1}{2} I^2 \int_0^L dL - (I^2 \int_0^M M \, dM)/L_s = \frac{1}{2} I^2L - \frac{1}{2} M^2I^2/L_s \tag{12}$$

The first term of Eq. (12) is identical to Eq. (2) and represents the magnetic energy stored in the field of a SRG. The second term corresponds to a decrease in the magnetic energy of the superconducting coil.

The differential work required of the energy source dW_s using Eq. (11) is

$$dW_s = I^2 \, dL + I_sI \, dM - MI^2 \, dM/L_s \tag{13}$$

In order to integrate Eq. (13), we obtain an expression for the superconducting current I_s as a function of I, L_s, and M by integrating Eq. (11) for constant current I, viz.

$$I_s \, (M) = I_{so} - IM/L_s \tag{14}$$

Here I_{so} is the initial superconducting current and M the mutual inductance during the launch, i.e., $M = M'x$ where x is the displacement along the rail. Using Eqs. (11) and (14) with Eq. (13), we obtain at the end of the launch ($M = M'\ell$) for the energy required from the energy source,

$$W_s = LI^2 + MII_{so} - M^2I^2/L_s \tag{15}$$

The mechanical work imparted to the projectile W_M is

$$W_M = W_s - W_m = \frac{1}{2} LI^2 + MII_{so} - \frac{1}{2} M^2I^2/L_s \tag{16}$$

This calculation has been carried out to the point at which the projectile leaves the rails, but it is important to note that the superconducting coil will recover energy from the magnetic field of the SRG to return to its prelaunch condition. For that reason we have not included the constant term $(1/2)L_sI_{so}^2$ in Eqs. (12) and (15). The recovered energy E_R is just the second term in Eq. (12), neglecting fluxon motion losses, i.e.,

$$E_R = \frac{1}{2} M^2I^2/L_s \tag{17}$$

Thus the energy lost in the muzzle resistor is substantially reduced. An evaluation of this energy saving will be made in a later section.

The superconducting augmented rail gun system, in addition to providing significant advantages in launch efficiencies which will be discussed in the next section, provides the benefits of charging before launching, retention of its stored energy between launches, and lessening the severity of field collapse after launch by recovery of a portion of the collapsing magnetic field energy of the rails.

The main disadvantage of a superconducting augmentation coil, i.e., cryogenic cooling will be discussed in a separate report. Based on the experience of the present authors in cryogenic technology, these losses which are not negligible are nonetheless smaller than the benefits obtained from the use of superconducting augmentation coils.

COMPARISON OF VARIOUS IDEAL RAIL GUN CONFIGURATIONS

For this comparison, we will assume that the currents in the augmentation coil and the rail current are equal.

$$I = I_A \quad \text{(normal augmentation)} \tag{18a}$$

$$I = I_{so} \quad \text{(superconducting augmentation)} \tag{18b}$$

Furthermore, we will use the expressions,

$$M = k(LL_A)^{1/2} \quad , \quad (k < 1) \tag{19a}$$

$$M = k(LL_s)^{1/2} \quad , \quad (k < 1) \tag{19b}$$

where k is the coefficient of magnetic coupling, in evaluating the systems.

Simple Rail Gun

The source must provide (Eqs. (2) through (4))

$$W_s = W_m + W_M = \frac{1}{2} I^2L + \frac{1}{2} I^2L = I^2L \tag{20}$$

Whereas the energy lost in the resistor is the total magnetic energy

$$W_m = \frac{1}{2} I^2L \tag{21}$$

Thus, the launch efficiency, defined as the ratio of the mechanical work to the source work, of the ideal SRG is 50 percent. Actual efficiencies will be somewhat less due to resistive heating (Joule) of the rails.

Rail Gun With Normal Conducting Augmentation

The sources must provide, using Eqs. (7), (18a), and (19a)

$$W_s = I^2L(1 + 2k\sqrt{L_A/L}) \tag{22}$$

and the mechanical energy is from Eq. (8)

$$W_M = \frac{1}{2} I^2L(1 + 2k\sqrt{L_A/L}) \tag{23}$$

An equal amount of energy is stored in the magnetic field and lost at the end of the launch.

Thus, the launch efficiency of the ideal rail gun with normal conducting augmentation is also 50 percent. Joule losses in the rail circuit are reduced as compared to the SRG because the same launch energy can be achieved with lower currents. However, this gain is offset by Joule heating losses in the augmentation coils. Furthermore, in a non-ideal system the magnetic field energy $(1/2)L_AI_A^2$ stored in the augmentation coil will be lost after each launch and must be resupplied from the power source for the next launch.

Rail Gun With Superconducting Augmentation Coils

The source provides (during each launch and after charging the superconducting coil with $(1/2)L_sI_{so}^2$ self inductance energy which is recovered) from Eqs. (15), (18b), and (19b)

$$W_s = LI^2(1 - k^2 + k\sqrt{L_s/L}) \tag{24}$$

The mechanical energy is using Eqs. (16), (18b), and (19b)

$$W_M = \frac{1}{2} LI^2(1 - k^2 + 2k\sqrt{L_s/L}) \tag{25}$$

And the magnetic field energy lost at the end of launch is (Eqs. (12) and (19b))

$$W_m = \frac{1}{2} LI^2(1 - k^2) \tag{26}$$

Assuming for purposes of discussion that $k = 0.5$ and $L_s = L$, the launch efficiency of the ideal rail gun with superconducting augmentation coils becomes 70 percent. It should be noted that the values assumed for k and L_s above represent conservative estimates of what can be achieved in practice.

An alternate design would be to place a comparatively large superconducting inductance in series with the augmentation coil in a configuration such that this "ballast" inductor does not see the strong magnetic field of the rails. In this case we have the condition

$$L_s^* \gg L \sim M \tag{27}$$

where L_s^* is now the total inductance of the augmentation circuit, coil plus ballast, and L and M are defined as before. This situation differs from the previous case in that there is weak total flux linkage between the rail and the augmentation circuit. Using Eq. (19b) and neglecting terms of the order M/L_s^*, Eqs. (24) through (26) are replaced, respectively, by

$$W_s = LI^2(1 + M/L) \tag{28}$$

$$W_M = \frac{1}{2} LI^2(1 + 2M/L) \tag{29}$$

$$W_m = \frac{1}{2} LI^2 \tag{30}$$

Assuming $M = L$, a condition which should be realizable without much difficulty with a pair of augmentation coils, the launch efficiency of the ideal rail gun with weakly flux linked superconducting augmentation is 75 percent. As can be seen by comparison with the above calculation, the launch efficiencies of the two superconducting designs are comparable.

However, in the first approach the acceleration of the projectile is largest at the beginning of the launch, the second approach provides for constant acceleration (for constant rail current) along the entire length of the rail, but at the expense of needing an additional coil. It would be interesting to explore the possible effects of linking this coil to the flux of the energy storage coil.

In selecting a final design for a superconducting augmentation system, the effects of "training"[5] of the superconducting coil(s) in the time dependent rail field will have to be carefully evaluated. In this preliminary study we have not considered the effects of training although available technology in design and materials will allow the virtual elimination of deleterious training.[5]

In selecting materials which minimize training effects, the commercially available Nb_3Sn conductors show virtually no training effects.[5] However, Nb_3Sn is extremely brittle and may require substantial support structures to withstand the effects of high g loading in an augmented rail gun system. We note that the new $Pd_xCu_{1-x}H$ technology developed jointly at Benet Weapons Laboratory and the State University of New York at Albany may be useful in rail gun technology.[6] The superconducting transition temperatures and critical current characteristics of the new $Pd_xCu_{1-x}H$ superconductors are comparable with Nb_3Sn. In addition, the new materials are ductile. Further

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[5]Saint-James, D., Sarma, G., and Thomas, E. J., Type II Superconductivity, Pergamon Press, 1969, p. 264.
[6]Leiberich, A., Scholz, W., Standish, W., and Homan, C. G., "Superconductivity in H Charged Cu Implanted Pd," Phys. Lett. 87A, 57 (1981).

research on PdCuH superconducting materials is planned under the AH60 program to evaluate the other useful properties of these new materials.

COMPARISON OF TWO ACTUAL RAIL GUN SYSTEMS

In the previous section we have discussed the ideal launch efficiencies of various rail gun systems. Joule heating losses in the rail and energy losses from the energy storage system after each launch were omitted from the discussion. Because the same launch velocities are achieved with lower rail currents in an augmented rail gun, additional energy savings can be attained in an augmented as compared to a simple rail gun.

These savings may not be realizable with normal augmentation because of the additional energy losses in the normal conducting augmentation circuit during each launch. However, in a superconducting augmented system these savings are real provided cooling energy requirements are kept small.

For purposes of discussion, we will compare the simple air defense rail gun[1] with a superconducting augmented air defense rail gun system. A coupling of $k = 0.5$ and $L = L_s$ has been assumed in Eqs. (24) and (25), resulting in an ideal launch efficiency of 70 percent for the superconducting augmented rail gun.

Table I gives typical energy requirements for the two configurations per burst of 20 shots in megajoules (MJ). For the superconducting augmented rail gun, the current has been scaled so that Eq. (25) results in the same kinetic

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[1]McNab, I. R. and Deis, D. W., "Study of an Electromagnetic Gun System For Air Defense," Westinghouse Research Center, November 1981. Final report prepared for the Air Force Armament Laboratory, Air Force Report No. AFATL-TR-81-99.

energy as Eq. (4) for the simple rail gun. Field energy lost from the rail is then calculated with the help of Eqs. (21) and (26), and Joule losses and field energy lost from the energy storage coil after each burst are calculated assuming scaling with I^2. (These estimates are conservative due to the assumption that $L = L_s$.) Estimates of cooling energy requirements have been omitted from Table I at this time because they require more precise assumptions about actual parameters. However, cooling is already required for the energy storage coil of the SRG at the rate of six liters of liquid nitrogen per burst. Since the air defense gun would require a burst every two minutes, the cooling requirement of this SRG would be 180 liters per hour of operation. Thus, we believe that the refrigeration requirements of the superconducting augmentation coils could be provided without much difficulty out of reduced cooling requirements for the energy storage coil due to lower currents.

Table I shows that substantial energy savings and increases in actual launch efficiencies could be realized by just adding a superconducting augmentation coil to an otherwise unchanged simple rail gun. Such energy savings and attendant reductions in rail currents would lessen the design requirements on the energy storage coil and the homopolar generator. Alternatively, existing systems could be made to achieve substantially higher launch velocities with such augmentation.

TABLE I. ENERGY REQUIREMENTS PER BURST FOR A SIMPLE RAIL GUN (SRG) AND A SUPERCONDUCTING AUGMENTED RAIL GUN (SCARG)

	SRG*	SCARG*
Kinetic Energy of Projectiles	3.04	3.04
Ideal Launch Efficiency	50%	70% (k=0.5)
Field Energy Lost from Rail Field**	3.04	1.30
Rail Current (Megaamp)***	0.578	0.437
Field Energy Lost from Energy Storage Coil	0.46	0.26
Joule Losses	3.44	1.97
Total Energy Required per Burst	9.98	6.57
Actual Launch Efficiency	30.5%	46.3%

*All energies given in megajoules (MJ).
**Calculated assuming constant rail current.
***Rail current calculated to yield the same launch velocities.

CONCLUSIONS

We have analyzed the effect of including superconducting augmentation as an adjunct system to existing rail guns using an energy approach.

By including the unique physical properties of superconductors, we have shown that such a superconducting augmented system manages the launch energies more efficiently as compared to a system based on normal conductors. Even with conservative parameters for a superconducting system, overall launch efficiencies are 50 percent higher than for a normal conducting systems.

Inclusion of superconducting augmentation coils therefore allows a greater range of parameters for the weapons system designers. Thus

significant savings in weight and size may be achieved in the case of a single use weapon system or increased projectile performance may be achieved in a system designed for multiple zones of use.

Finally, once superconductivity is adopted as a design parameter, other sources of energy savings or weight and size reductions could be found in a particular EMG system. A particular example of this type of application, resulting in enhanced performance, is the use of superconducting field excitation coils in the homopolar generator in order to overcome the saturation effects associated with the use of magnetic materials.

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APPENDIX

VARIABLE CURRENT LAUNCHERS

In this appendix we will consider the situation of variable rather than constant current in the launcher rails. The system configuration to be analyzed will consist of a charged inductor coil, normally contained in the energy source for pulse shaping purposes, feeding the rails. This approach will also allow the derivation of the projectile driving force from the magnetic potential energy rather than from the kinetic energy of the projectile as is required in the approach taken in the main body of the report.

In this analysis, the current I varies and we introduce the driving coil inductance L_o in series with the rail inductance L.

We first consider the case of a simple rail gun.[4] Kirchoff's law applied to the inductor L_o driving the rails yields,

$$L_o \frac{dI}{dt} + \frac{d}{dt}(xIL') + (R+R'x)I = 0 \qquad \text{(A1)}$$

where R is the coil resistance, L' and R' are the rail inductance and resistance per unit displacement of the slider, and x is the displacement of the slider along the rails. Neglecting rail and inductor resistance and making use of $d/dt = (d/dx)(dx/dt)$ we obtain

$$L_o \frac{dI}{dx} + \frac{d}{dx}(xIL') = 0 \qquad \text{(A2)}$$

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[4]McNab, I. R. and Deis, D. W., "Study of an Electromagnetic Gun System For Air Defense," Westinghouse Research Center, November 1981. Final report prepared for the Air Force Armament Laboratory, Air Force Report No. AFATL-TR-81-99, Appendix A, p. 137.

which can be solved immediately using the proper initial condition for I to yield

$$I = \frac{L_o}{1 + x(L'/L_o)} \tag{A3}$$

The total magnetic energy of the coil-rail system as a function of projectile position is given by

$$W_{mT} = \frac{1}{2} L_o I^2 + \frac{1}{2} xL'I^2 \tag{A4}$$

Inserting Eq. (A3) into Eq. (A4) and differentiating we obtain the force

$$F_x = - \frac{dW_{mT}}{dx} = - (L_o+xL')\ I \frac{dI}{dx} - \frac{1}{2} L'I^2 = + \frac{1}{2} L'I^2 \tag{A5}$$

This result agrees with the force one obtains by taking the <u>positive</u> derivative of the kinetic energy with respect to x in Eq. (4) even though Eq. (4) was derived for the condition of constant current.

We now include superconducting augmentation in the system. Kirchoff's law for the coil-rail system, neglecting again the resistances of the coil and rails, and after changing the differentiation from t to x now reads

$$L_o \frac{dI}{dx} + \frac{d}{dx}(xIL') + \frac{d}{dx}(xI_sM') = 0 \tag{A6}$$

and for the superconducting augmentation coil,

$$\frac{d\phi_s}{dx} = L_s \frac{dI_s}{dx} + \frac{d}{dx}(xM'I) = 0 \tag{A7}$$

where M' is the mutual inductance per unit length between the rail and the superconducting coils.

Eq. (A7) can be integrated directly to yield

$$I_s = I_{so} - \frac{xIM'}{L_s} \qquad \text{(A8)}$$

using the initial condition for I_s. By inserting Eq. (A8) into Eq. (A6), the differential equation uncouples and we obtain

$$\left(1 + x\frac{L'}{L_o} - x^2\frac{M'^2}{L_oL_s}\right)\frac{dI}{dx} + \left(\frac{L'}{L_o} - 2x\frac{M'^2}{L_oL_s}\right)I = -\frac{I_{so}M'}{L_o} \qquad \text{(A9)}$$

The analytical solution to this equation, using the initial condition on I, is

$$I = \frac{I_o - xI_{so}(M'/L_o)}{1 + x(L'/L_o) - x^2(M'^2/L_oL_s)} \qquad \text{(A10)}$$

The correctness of this solution can be checked by insertion in Eq. (A9). Using this solution in Eq. (A8) gives the dependence of the current in the superconducting augmentation coil as a function of the slider position.

The total magnetic energy of the coil-rail system, W_{mT}, including the superconducting augmentation coil is now given as a function of x by

$$W_{mT} = \frac{1}{2}L_oI^2 + \frac{1}{2}xL'I^2 + \frac{1}{2}L_sI_s{}^2 + xII_sM' \qquad \text{(A11)}$$

Eliminating I_s with the help of Eq. (A8) leads to

$$W_{mT} = \frac{1}{2}L_oI^2 + \frac{1}{2}xL'I^2 + \frac{1}{2}L_sI_{so}{}^2 - \frac{1}{2}\frac{(xIM')^2}{L_s} \qquad \text{(A12)}$$

The third term in this equation represents the initial magnetic energy of the superconducting augmentation coil. Since it is a constant, it does not contribute to the force and has been omitted in the main text. Inserting the expression for I from Eq. (A10) and differentiating, one obtains for the force after a somewhat lengthy but straightforward calculation

$$F_x = - \frac{dW_{mT}}{dx} = \frac{1}{2} L'I^2 + II_{so}M' - xI^2 \frac{M'^2}{L_s} \qquad (A13)$$

Equation (A13) is identical to the result one obtains by taking the positive derivative of the kinetic energy in Eq. (16) with respect to the slider position. Thus, as expected, the forces derived from the expressions in the main text assuming constant rail current agree with the ones derived here for variable currents, provided the instantaneous current is used in these expressions.

It is important to realize that it is only proper to differentiate the total magnetic energy to derive the force if one is dealing with a conservative system. That is, the magnetic circuit must be decoupled from sources that provide a continuous energy input and the time dependence of the current must be taken into account. This was not the approach taken in the main text and thus the force there should be determined by differentiation of the kinetic energy expressions.

The energy expressions in the main text represent integrals over the force assuming constant currents. Thus, they need to be modified in the case where the situation of variable currents pertains. From conservation of energy, the mechanical kinetic energy, W_M, is given by the difference of the total magnetic energy before and after launch (Eqs. (A4) and (A12)). The change in magnetic energy of the inductor coil, corresponding to the source work, W_s, in the main text, can be evaluated using Eqs. (A3) and (A10) for the current at the beginning and the end of launch. Finally, the magnetic energy stored in the rail system at the end of the launch, which will be dissipated in the muzzle resistor, is obtained by forming the difference between W_s and

W_M, i.e., by omitting the inductor coil term $(1/2)L_oI^2$ in Eqs. (A4) and (A12) and calculating the appropriate change in magnetic energy. In as much as these expressions will be rather involved, at least in the case of the rail gun with superconducting augmentation, we have chosen not to present them here. However, it is worth mentioning that since the magnetic energy stored in the rail system depends only on the rail current at the end of the launch, decreasing currents during the launch as in Eqs. (A3) and (A10) will generally lead to increased launch efficiencies as compared to the situation of constant rail current. Since this applies to the simple rail gun as well as to the rail gun with superconducting augmentation, the conclusions drawn in the main text regarding efficiencies remain essentially unaffected.

The above comment regarding launch efficiencies may be somewhat irrelevant since, in the design of real rail guns, the primary objective would be the attainment of maximum kinetic energy for a rail of given length. To accomplish this purpose, the designers would opt to keep the rail current as high and as constant as possible throughout the launch. Thus the analysis performed in the main text represents the appropriate ideal limiting situation for practical designs.

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